

Q.1. If a, b, c are three elements of a group $(G, \circ)$ then

(I) $a \circ c = b \circ c \Rightarrow a = b$ (Right Cancellation Law)

(II) $c \circ a = c \circ b \Rightarrow a = b$ (Left Cancellation Law)

Proof: I $a \circ c = b \circ c \Rightarrow (a \circ c) \circ c^{-1} = (b \circ c) \circ c^{-1}$ [$\because c^{-1} \in G$]
 $\Rightarrow a \circ (c \circ c^{-1}) = b \circ (c \circ c^{-1})$ (By associativity)
 $\Rightarrow a \circ e = b \circ e \Rightarrow a = b$

II $c \circ a = c \circ b \Rightarrow c^{-1} \circ (c \circ a) = c^{-1} \circ (c \circ b)$
 $\Rightarrow (c^{-1} \circ c) \circ a = (c^{-1} \circ c) \circ b$
 $\Rightarrow e \circ a = e \circ b \Rightarrow a = b$

Q.2 In a group G , the equation $a \circ x = b$ and $y \circ a = b$ where $a, b \in G$ have unique solution in G .

Proof: $a \circ x = b$
 $\Rightarrow a^{-1} \circ (a \circ x) = a^{-1} \circ b$ [$\because a^{-1} \in G$]
 $\Rightarrow (a^{-1} \circ a) \circ x = a^{-1} \circ b$ (By associativity)
 $\Rightarrow e \circ x = a^{-1} \circ b$
 $\Rightarrow x = a^{-1} \circ b \in G$ [$\because a^{-1}, b \in G \Rightarrow a^{-1} \circ b \in G$]

Therefore, the equation $a \circ x = b$ has a solution $x = a^{-1} \circ b$ in G .
 Similarly, it can be proved that the equation $y \circ a = b$ has a solution $y = b \circ a^{-1}$ in G .

Uniqueness:- Let x_1 and x_2 be any two solutions of the equation

$a \circ x = b$, so that

$a \circ x_1 = b$ and $a \circ x_2 = b$

$\Rightarrow a \circ x_1 = a \circ x_2$

$\Rightarrow x_1 = x_2$ (By left Cancellation Law)

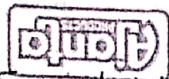
Therefore, the solution of the equation $a \circ x = b$ is unique.

Similarly, it can be proved that the equation

$y \circ a = b$

Hence, the given equations have unique solutions in G .

Proved



Q.3. The left identity is also the right identity.

Proof: Let e be the left identity of a group G and let $a \in G$ be any element. Then

$$ea = a \quad \text{--- (1)}$$

To prove that e is also the right identity, it is sufficient to show that

$$ae = a \quad \text{--- (2)}$$

Let a^{-1} be the inverse of a , then

$$a^{-1}a = e \quad \text{--- (3)}$$

$$\text{Now, } a^{-1}(ae) = (a^{-1}a)e = e \cdot e = e = a^{-1}a$$

$$\Rightarrow a^{-1}(ae) = a^{-1}a \quad (\text{By left cancellation law})$$

$$\Rightarrow ae = a \quad (\text{By left cancellation law})$$

Hence, we have that

The left identity is also the right identity of G .

proved

Q.4 The left inverse of an element is also its right inverse.

Proof: Let a^{-1} be the left inverse of an element a of a group G , so that

$$a^{-1}a = e \quad \text{--- (1)}$$

where e is the identity of G

We have to prove that a^{-1} is also the right inverse of a , it is sufficient to show that

$$aa^{-1} = e \quad \text{--- (2)}$$

By associativity, we have

$$a^{-1}(aa^{-1}) = (a^{-1}a)a^{-1} = ea^{-1} \quad [\text{by (1)}]$$

$$= a^{-1} = a^{-1}e$$

$$\Rightarrow a^{-1}(aa^{-1}) = a^{-1}e$$

$$\Rightarrow aa^{-1} = e \quad (\text{By left cancellation law})$$

Hence, from (2), we can say that the left inverse of an element is also the right inverse of that element.

Proved.